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Spatial velocity profile changes along the cord in normal human fetuses: can these affect Doppler measurements of venous umbilical blood flow?

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KEYWORDS: Doppler flow measurement; umbilical venous circulation; velocity spatial distribution

ABSTRACT

Objective *Several studies have assumed a parabolic velocity profile through the umbilical vein (UV) to derive the mean spatial velocity that is indispensable for flow rate calculations. However, the structure and arrangement of the umbilical cord suggest that velocity profiles may vary. The aim of this study was to evaluate UV spatial flow velocity profiles at different sites along the umbilical cord.*

Methods *Ten singleton pregnancies with a gestational age between 26 and 34 weeks were included in the study. Ultrasound equipment with an inbuilt function for analysis of the spatial velocity profile along a line located in a fixed plane was used to obtain UV velocity profiles. Velocity profiles were obtained at the placental insertion and in a free intra-amniotic loop of the cord. Two-dimensional (2D) velocity distribution coefficients were evaluated as ratios between mean and maximum velocities along the investigated lines.*

Results *2D velocity distribution coefficients at the placental insertion (0.85 ± 0.03) were significantly higher ($P < 0.00001$) than those obtained from a free loop of cord (0.76 ± 0.03). Values indicated that velocity profiles are approximately flat at the placental insertion and become more parabolic moving downstream. Moreover, profiles become skewed in association with cord curvature and show peculiar biphasic shapes immediately downstream from the placenta.*

Conclusions *Flow velocity profiles in the UV are not perfectly parabolic and modify along the cord. These characteristics may affect the evaluation of UV blood flow rate. Copyright © 2003 ISUOG. Published by John Wiley & Sons, Ltd.*

INTRODUCTION

Volumetric blood flows in vessels are important indicators of human pathophysiological conditions. In particular, a quantitative knowledge of human fetal blood flow patterns is of major clinical importance in assessing possible fetal cardiocirculatory diseases; ultrasound is the only non-invasive technique for such measurements *in utero*.

Even if, conceptually, the measurement of blood flow by the Doppler technique is relatively simple, in practice several potential sources of error exist[1–3]. The reliability of this method for measuring the blood flow rate depends on the accuracy of measuring the vessel diameter and of obtaining the mean spatial velocity value at the same site. Another critical issue of Doppler velocimetry concerns the angle between the ultrasound beam and the direction of blood flow; values below 30° are recommended to obtain an accurate velocity evaluation.

Digital high-resolution imaging and color Doppler visualization facilitate measurements of vessel diameter and beam-vessel angles, however, questions concerning the evaluation of the mean spatial velocity persist.

Different methods have been suggested to quantify the mean spatial velocity, i.e. the mean of the blood velocities in a given cross-section of the investigated vessel at a fixed time. Ideally, the mean spatial velocity could be calculated as intensity-weighted mean velocity (IWMV) directly by conventional pulsed Doppler equipment, provided that the sample volume covers the entire cross-section, but this value is generally highly unstable because of noise due to echoes produced by neighboring vessels and wall movement[2–4]. Furthermore, the effects of the adopted high-pass filters on IWMV are unpredictable.

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Generally, the mean spatial velocity is deduced from the maximal velocity measured in the investigated area, scaled by a spatial velocity distribution coefficient. In such cases, a detailed knowledge of the hemodynamics within the insonated vessel cross-section is required. Cross-sectional information of the spatial velocity profiles may be obtained *in vivo* both by using sophisticated three-dimensional (3D) color[5,6] or multigate-spectral technologies[7]. Furthermore, flow velocity profiles due to complex fluid dynamics may be simulated and investigated by advanced computational model simulations[8,9]. Each technique is potentially effective but demands a specific validation for an individual vessel.

These considerations particularly apply to the Doppler measurement of blood flow rate through the umbilical vein (UV) in human fetuses, which is a potential tool in elucidating the mechanisms of intrauterine growth restriction. Although several studies measured the umbilical venous flow in normal fetuses, extremely wide normal ranges were obtained. Whether this high data variability reflects a physiological variability or is mainly due to methodological errors has still to be ascertained. In fact, perfect parabolic profiles have been assumed to exist[10–13] throughout the cord vein or IWMV has been used[14,15] to estimate the mean spatial velocity in the UV.

The aim of the present study was to investigate the spatial flow velocity profiles through the UV in human fetuses; this analysis was performed at different sites along the umbilical cord in order to assess any possible differences.

METHODS

Theoretical considerations

According to its definition (the amount of fluid volume crossing an area in a unit of time), the volumetric flow rate in a vessel at a time t can be calculated as the product of the instantaneous mean velocity $V_{\mathrm{mean}}(t)$ in the considered cross-section and the cross-sectional area $CSA(t)$:

$$Q(t) = V_{\mathrm{mean}}(t) \cdot CSA(t) \quad (1)$$

Generally, the blood velocity in a vessel changes both in time and in space; in order to avoid any misunderstanding, we use the terms 'mean' and 'average' to indicate the spatially-averaged value of the velocity and the time-averaged value, respectively.

Equation 1 may be rewritten as:

$$Q(t) = h_{CSA}(t) \cdot V_{\max}(t) \cdot CSA(t), \quad (2)$$

where $h_{CSA}(t)$ indicates a spatial velocity distribution coefficient, calculated as the ratio between $V_{\mathrm{mean}}(t)$ and the instantaneous maximal velocity $V_{\max}(t)$ in the cross-section. $V_{\max}(t)$ values correspond to the maximum Doppler tracing and $CSA(t)$ may be evaluated on the basis of the measured vessel diameter, assuming a circular shape.

The $h_{CSA}(t)$ value depends on the spatial velocimetric profile in the cross-section and is highly related to the local hemodynamics: a value of 1.0 corresponds to a flat profile (i.e. a plug flow as at the entrance of a pipe from a reservoir), a value of 0.5 may be associated with parabolic flow profiles (i.e. a completely developed steady laminar flow in a circular straight tube) and lower still coefficients are related to a core of blood flowing at high velocity down the center of the vessel. Furthermore, $h_{CSA}(t)$ value may be time-varying and depend on the different phases of the cardiac cycle, but this influence is quite modest for the UV due to almost steady flow in normal conditions[6,10]. Hence, in case of UV flow a constant value h_{CSA} may be assumed during the whole cardiac cycle. Usually, a value of 0.5 is used for h_{CSA} in the umbilical venous flow calculation, under the hypothesis of parabolic flow velocity profiles[10–13]. However, hemodynamic theory suggests different velocity profiles at different distances from the placenta, as the boundary layer grows at the wall due to viscous effects[16]. Furthermore, we expected that the complex structure and arrangement of the umbilical cord would affect the shape of the flow velocity profiles.

Study group and measurements

Ten normal singleton pregnancies were included in this cross-sectional study. In all cases growth parameters and morphology were confirmed to be normal at birth. Color Doppler investigation at different sites of the UV was performed in all fetuses only once from 26 to 34 weeks of gestation. Ultrasound examination was performed with an Aloka Prosound SSD 5500 unit (Aloka, Tokyo, Japan) equipped with a 1.5–8-MHz multifrequency convex transducer (values of 3 or 3.75 MHz were used). Pulse repetition frequencies were in the range 4–4.5 kHz, the spatial-peak temporal-average intensities were < 100 mW/cm^2 and a 50-Hz high-pass filter was used. During color Doppler imaging the gray scale is clearly visible and allows the operator to exactly define the vessel walls and to set an optimal gain; the optimal gain setting was simply obtained by maximizing the gain level without random color noise in the non-flow areas (i.e. the color uniformly fills the vessel but is not seen in the adjacent walls[17]).

The ultrasound machine is equipped with a function for the analysis of the spatial velocity profile based on Doppler color flow mapping. The velocity profile may be displayed along an arbitrary line on a playback color Doppler image: a velocity value is associated with each pixel along the specified line according to different brightness of the image (Figure 1). High spatial resolution is achievable due to very small pixel dimensions (< 0.1 mm).

Different cross-lines were traced to obtain velocity information on different cross-sections of the UV. Namely, spatial flow velocity profiles were obtained close to the placental insertion of the cord (i.e. within 15 mm) and in a free intra-amniotic loop of the cord at a distance of at least 30 mm from the placental insertion. At least two measurements were performed

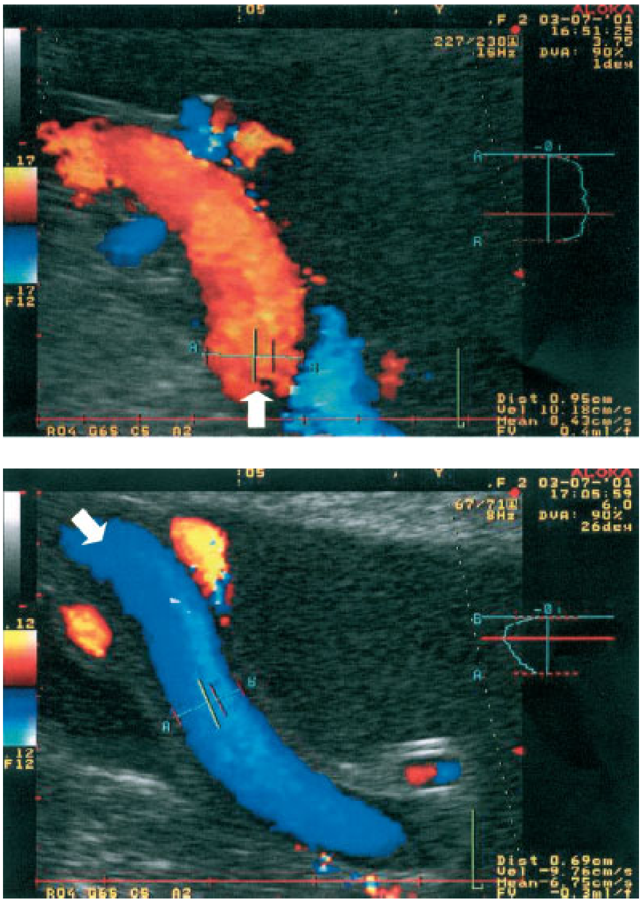

Figure 1 An example of the color Doppler flow profile analysis at the placental insertion (upper panel) and in an intra-amniotic free loop (lower panel) of the same umbilical vein. Flow velocity profiles were obtained along vessel diameters A–B. Reported values are not absolute values of the flow velocities. Dist, A–B distance; mean, mean of the recorded velocities; vel, velocity measured at the radial site indicated by the red line. The arrows indicate blood flow directions.

at each site; when possible, several velocity profiles were investigated at different positions on the same image (Figure 2).

The evaluation of h_{CSA}, which is indispensable for the calculation of flow rate (Equation 2), is not achievable on the basis of the recorded velocity profiles. Indeed, spatial velocity data are two-dimensional (2D) and were available only on the investigated plane but not for the cross-sectional points lying out of the plane. Nevertheless, a 2D velocity distribution coefficient h_{2D} was calculated as the ratio between the mean and maximum velocities recorded along the specified line on the examined plane. Even if the absolute velocity values measured on the color image are not reliable due to uncorrected beam-vessel angle, the h_{2D} values are calculated as velocity ratios. It is noteworthy that the coefficient h_{2D}, even if correlated with h_{CSA}, does not coincide with it (see Appendix). No reliable quantitative deduction of the h_{CSA} value can be made on the basis of purely 2D data.

Statistical methods

Shape coefficients h_{2D} calculated at different sites of the same UV were compared by paired *t*-test. The mean of two reliable measurements was used at each site of each cord. A statistically significant difference was defined as $P < 0.05$. Average data are expressed as mean ± SD.

RESULTS

In all fetuses the velocity profile was obtained at a cross-section of the UV at the placental insertion of the cord (at a distance of 5.8 ± 4.8 mm) and in an approximately straight segment of a free intra-amniotic loop of the cord. Figure 1 shows an example of the velocity profiles recorded at two different sites in the same UV: the velocity profile obtained close to the placental insertion is almost flat (upper panel) and that obtained from a free loop is more parabolic (lower panel).

The shape coefficients (h_{2D}) measured at the placental insertion were 0.85 ± 0.03 (minimum = 0.81, maximum = 0.9) and were significantly higher ($P < 0.0001$) than those obtained from a free loop (0.76 ± 0.03, minimum = 0.72, maximum = 0.81). This confirmed the above qualitative observation that velocity profiles vary at different sites in the cord.

Moreover, profiles skewed towards the outer wall in correspondence with cord curvature (Figure 2) and often showed a peculiar biphasic shape (camel-hump profile) immediately downstream from the placental insertion, probably related to lateral inflow of cotyledonar veins (Figure 3). Figure 4 shows several velocity profiles measured in the UV of the same fetus (upper panel): the velocity profiles are almost flat close to the placenta and become more parabolic moving downstream; as discussed above a biphasic velocity profile can be noted at the placental insertion. The flow profiles obtained *in vivo* were quite similar to those (lower panel) deduced from the fluid-dynamic theory[18] of fluid passing from a reservoir (i.e. the placenta) to a cylindrical tube (i.e. the UV).

DISCUSSION

Volumetric flow measurements based on analysis of the Doppler waveform require appropriate evaluation of the mean spatial velocity of blood flowing through the cross-section being investigated.

Two methods are available: the IWMV method, where mean spatial velocity is estimated directly from the total echo spectrum returned to the transducer, and the spatial velocity distribution coefficient method in which the highest velocity detected is multiplied by a coefficient reflecting the spatial distribution of velocities below the instantaneous maximum. Neither method is error free[2,3]. The IWMV method depends on factors such as the integrity of the conversion of echo-strength to digital values, a process not under the control of the user of the machine. Generally, measurement of maximal velocity is more clearly defined; the critical issue is then the estimate

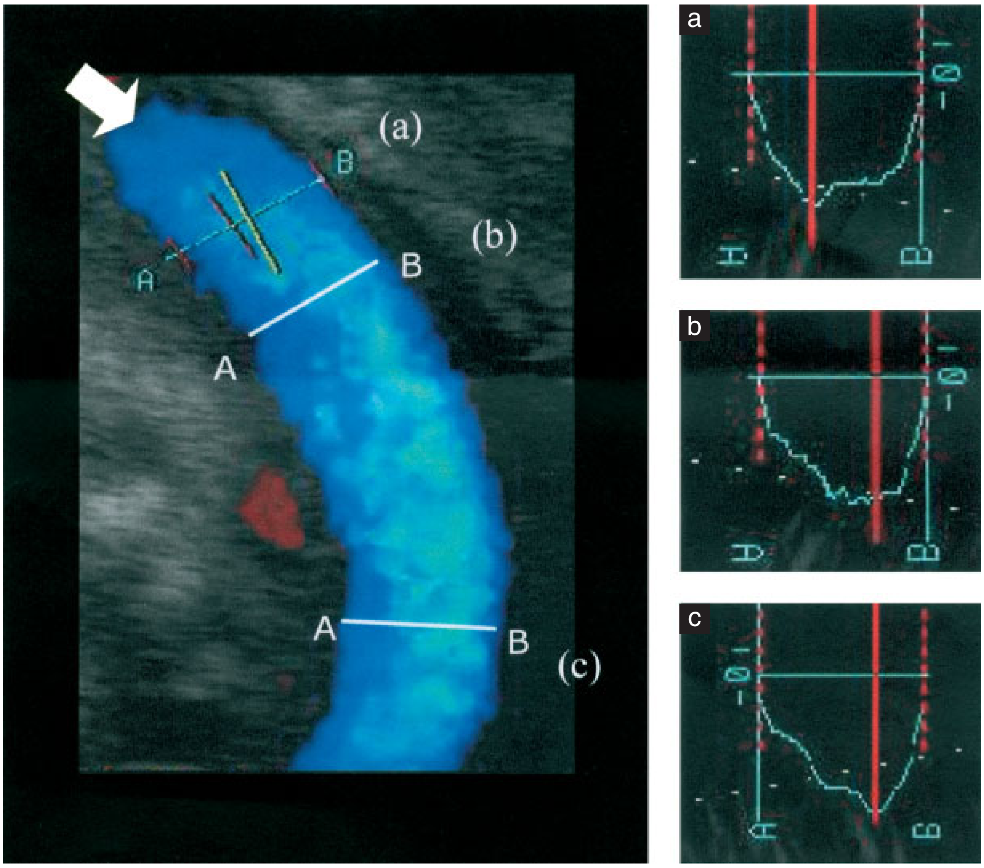

Figure 2 Velocity profiles in an intra-amniotic curved tract of the umbilical vein: the velocity profile at the entrance (a) shows maximum velocity close to the inner wall A (due to non-visible, upstream phenomena) and progressively (b and c) skews towards the outer wall B of the vessel due to inertial effects. The arrow indicates blood flow direction.

of an appropriate velocity distribution coefficient (h_{CSA}). From a theoretical point of view this coefficient ranges from 0 to 1, but values less than 0.3 are rare. It was assumed that at the ventricular outflow tracts, where the velocity profile is thought to be uniform across the vessel diameter (i.e. mean velocity equal to maximum velocity), a value equal to 1 would be obtained; however, several studies based on conventional 2D[19,20] or 3D color Doppler echocardiography[5] and magnetic resonance imaging[21] demonstrated markedly skewed profiles in the ascending aorta, with higher velocities towards the interventricular septum. This occurrence indicates that simple hemodynamic assumptions may be erroneous, even in well-investigated vascular areas, and that ideal spatial velocity distributions are at odds with the biological reality.

Similar considerations apply to the UV in human fetuses. A perfect parabolic profile is the hypothetical norm giving a velocity distribution coefficient of 0.5 for use in calculation of the flow rate. However, the present study indicates that flow velocity profiles in the UV are not perfectly parabolic and vary along the cord both due to wall (boundary layer development under viscous forces) and curvature (inertial phenomena) effects.

Hemodynamic features associated with complex anatomies cannot be deduced *a priori*, but need detailed analyses that may be achieved by means of *in vivo* sophisticated techniques[5–7], or animal[22] and computational models[8,9]. Conversely, caution should be applied when conventional 2D Doppler systems are used to quantitatively extrapolate 3D hemodynamic behavior. For example, color velocity data obtained in a single plane may be properly used to obtain information about the shape of the spatial velocity profiles (as shown in the present work on the human UV), but they are not adequate to quantify vessel flow rate[6]. Simple techniques based on the digital color Doppler velocity distribution from conventional 2D color Doppler were suggested for flow quantification[17] but rely on the restrictive assumption of axial symmetry of flow distribution[5]. Two recent studies[23,24] used digital color Doppler integration of 2D data to calculate the umbilical venous blood flow in human fetuses, obtaining higher values than those in previously published studies[10–15]. Although in those studies the same ultrasound equipment was used as in the present study, their results require review, since no reliable quantitative information about flow rate is achievable on the basis of purely 2D data.

According to the results of the present study, UV spatial velocity profiles modify with the site of Doppler insonation and it is impracticable to define a single coefficient h_{2D}, since it significantly decreases from 0.85 (0.81–0.9) to 0.76 (0.72–0.82) moving from the placental end to a free loop of the cord. Furthermore, h_{2D} values

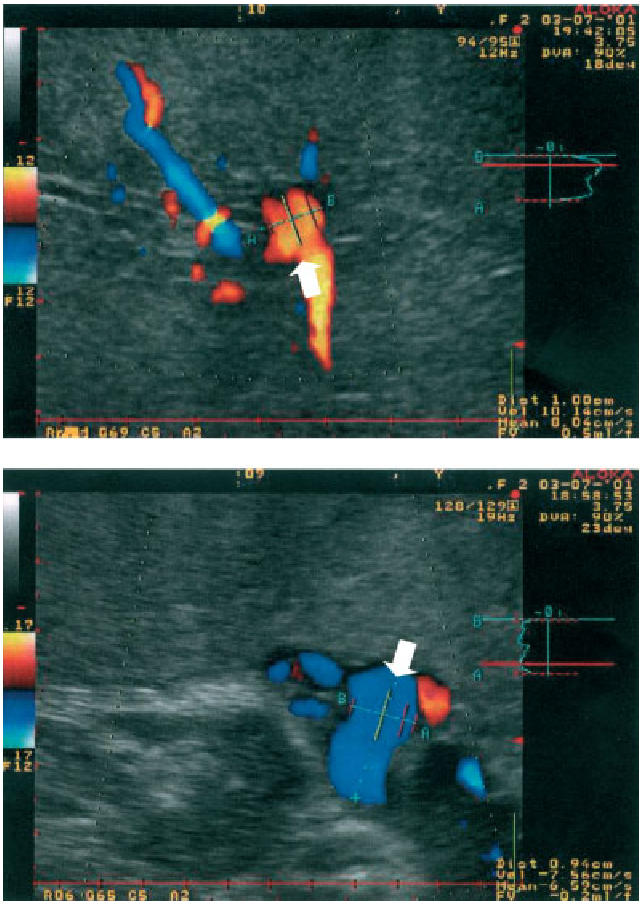

Figure 3 Two examples of the peculiar camel-hump velocity profiles often occurring at the placental insertion, probably related to the placental vessel inflows. The arrows indicate blood flow directions.

cannot be used, in place of h_{CSA} values, to scale the maximum Doppler velocity and calculate the blood flow rate through the UV (Equation 2). Theoretical values for the cross-sectional velocity distribution coefficient h_{CSA} may be obtained under the assumption of an axial-symmetric profile (see Appendix); in which case (but we do not know to what extent the assumption of axial-symmetrical profile is suitable for the UV velocity profiles) h_{2D} values of 0.85 and 0.76 would correspond to h_{CSA} values equal to 0.74 and 0.61, respectively. Although this latter value indicates an approximately parabolic profile, the classical use of an 'ideal' coefficient of 0.5 would create an 18% underestimation of UV flow rate in a free loop of the cord. Greater underestimations would occur for Doppler velocity recordings closer to the placenta.

Conversely, overestimations of the actual flow rate are generally expected when the IWMV method is used. Indeed, even if the sample volume covers entirely the vessel lumen in the insonated plane, only few flow velocities outside of the plane are included in the IWMV evaluation: this roughly corresponds to using h_{2D}, which is normally greater than h_{CSA}. Furthermore, filtering of the lowest velocity at the vessel wall could further augment the IWMV.

In conclusion, this study suggests that the extremely wide normal ranges of UV flow rate observed in normal fetuses, as well as the high variability between the results of various authors[10–15], may be partially explained by the varying venous velocity profiles along the umbilical cord. Further investigations are necessary to examine the 3D velocity profiles through the UV in a larger group of human fetuses and define accurate h_{CSA} values.

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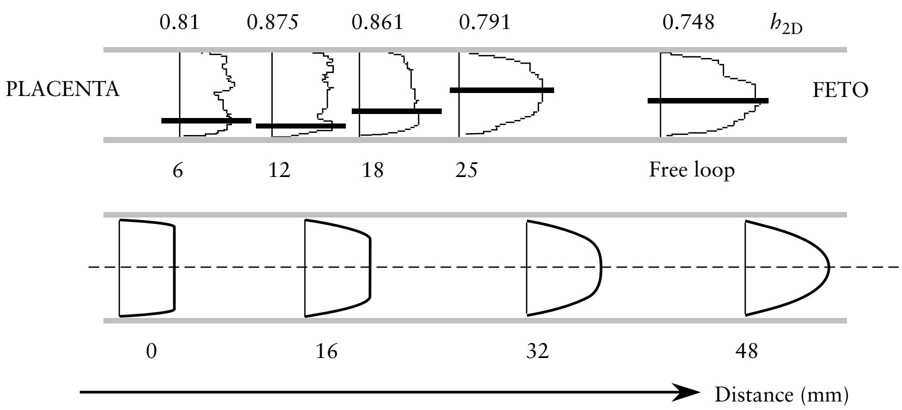

Figure 4 Comparison between the umbilical vein (UV) velocity profiles recorded at different distances from the placental insertion in the same fetus (upper panel) and those (lower panel) deduced from the fluid-dynamic theory of a fluid passing from a reservoir (i.e. the placenta) into a cylindrical tube (i.e. the UV).

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APPENDIX

The relationship between the two coefficients, h_{2D} and h_{CSA}, can be assessed for an axial-symmetric flow across a circular cross-section. In this case the velocity distribution may be described in a general form:

$$v(r) = V_{\max}\left(1 - \left(\frac{r}{R}\right)^n\right), \tag{A1}$$

where r is the distance between the point of interest and the center of the circular section and n is a parameter indicating the bluntness of the flow velocity profile. When $n = 2$, the flow has a perfect parabolic distribution, while a plug flow is obtained for very high values of n (perfect flat profile for $n \to \infty$). The coefficients h_{CSA} and h_{2D} can be calculated as ratios between the maximal and a mean spatial velocity, evaluated considering the whole three-dimensional profile ($\overline{V}_{CSA}$) or only its intersection with a plane through the axis ($\overline{V}_{2D}$), respectively. In turn, the mean spatial velocities are calculated as

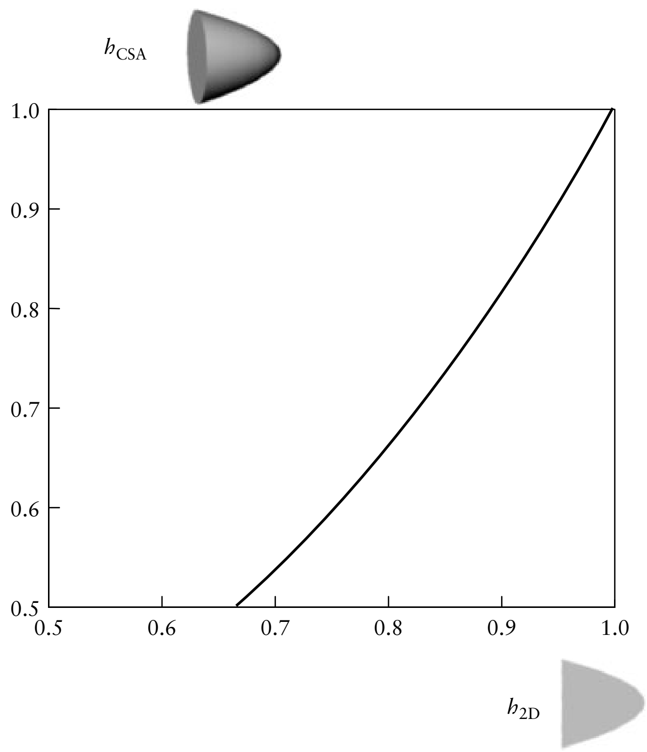

Figure 5 Relationship between the two-dimensional (h_{2D}) and the true, three-dimensional (h_{CSA}) spatial velocity coefficients in the case of an axial symmetric flow profile.

ratios between the spatial velocity integrals and the integration spaces:

$$\overline{V}_{CSA} = \frac{\int_0^R v(r)\cdot 2\pi r\cdot dr}{\pi R^2} \qquad \overline{V}_{2D} = \frac{2\int_0^R v(r)\cdot dr}{2R}, \quad \text{(A2)}$$

hence

$$\begin{aligned} h_{CSA} &= \frac{\overline{V}_{CSA}}{V_{\max}} = \frac{1}{V_{\max}\pi R^2}\int_0^R v(r)\cdot 2\pi r\cdot dr \\ &= \frac{2}{R^2}\int_0^R \left(1-\left(\frac{r}{R}\right)^n\right)\cdot r\cdot dr = \frac{n}{n+2} \quad \text{(A3)} \end{aligned}$$

$$\begin{aligned} h_{2D} &= \frac{\overline{V}_{2D}}{V_{\max}} = \frac{2}{V_{\max}2R}\int_0^R v(r)\cdot dr \\ &= \frac{1}{R}\int_0^R \left(1-\left(\frac{r}{R}\right)^n\right)\cdot dr \\ &= \frac{n}{n+1}. \quad \text{(A4)} \end{aligned}$$

According to these calculations, it is evident that h_{2D} is higher than h_{CSA} for any n finite value and that their difference decreases for increasing n (Figure 5).

